Chern-Weil theory for quasi-isomorphisms

Chern-Weil theory was used by Shulman (Berkeley thesis, 1972) to give explicit closed simplicial differential forms on the simplicial manifold $B_{\cdot}GL(n)$, which realize the Chern classes. For example, the only nonzero component of the differential form $c_1$ is

$$\text{Tr}(g^{-1}dg) \in A^1(GL(n)),$$

and the fact that it defines a closed simplicial form on $B_{\cdot}GL(n)$ amounts to the multiplicativity of the determinant. Likewise, there are two nonzero components of $c_2$, namely

$$\frac{1}{2}\text{Tr}((g^{-1}dg)^3) \in A^3(GL(n))$$

and

$$\frac{1}{2}\text{Tr}(g_1^{-1}dg_1)(dg_2g_2^{-1})) - \text{Tr}(g_1^{-1}dg_2)\text{Tr}(g_2^{-1}dg_2) \in A^2(GL(n) \times GL(n)),$$

and the fact that the simplicial differential form that these forms comprise is closed is known as the Polyakov-Wiegmann formula.

In this talk, I will define an extension of these forms to the classifying stack of perfect complexes (that is, extend these formulas to the case where $g$ is a quasi-invertible map between finite-dimensional complexes). These extensions were proved to exist by Toën and Vezzosi, but the explicit formulas are new. In the case of $c_1$, we obtain a new perspective on the determinant of quasi-invertible maps, defined by Knudsen and Mumford.