Bacteria are among the oldest and most abundant living species on Earth, and their activity influences the planet’s environmental dynamics in multiple ways. Bacteria often migrate en masse over large distances, moving in dense groups in a highly organized, collective fashion known as “swarming motility.” The flow dynamics of dense bacterial colonies can be very complex and, because of the interaction between the bacteria and the fluid, remarkably different from those predicted by conventional fluid models. In particular, turbulent swimming patterns often emerge, characterized by chaotic motions and the formation of vortices, even in situations where liquids should exhibit laminar flow. But these complex phenomena are difficult to characterize experimentally, and a predictive model that describes them has not emerged to date. In my talk I will survey the most recent progress in experimental and theoretical studies of collective swimming of bacteria suspended in Newtonian or anisotropic liquids [1,2].